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## DENSITY OF BIDISPERSE POWDER MATERIALS

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Computational formulas for determination of the density of bidisperse powder materials are proposed that take into account the effect of the ratio between the sizes of the material particles. The derived formulas are recommended for engineering calculations in developing various mixtures.

The packing of grog grains and filler granules, the structure of composite materials, etc. are evaluated using the relative bulk density  $V_r$ . The bulk density has great significance in solving various technological problems related to the production of porous and composite materials and articles [1, 2], which implies the usefulness of a theoretical evaluation of it.

To calculate the density of freely poured bidisperse powder materials, let us assume that the large particles with an average size  $D$  form a volume lattice with a relative bulk density  $V_{r,c}$  (if the latter is not known, we take the density of random packing, equal to 0.63).

In pores the small particles with a size  $D_m$  are distributed partially or completely among large ones. Large particles ( $D D_m$ ) distant from the surface are distributed in correspondence with a bulk density  $V_{r,m}$  (or with a density of 0.63).

We assume the elementary cell of the considered system to be a rectilinear triangular prism whose edges are equal to  $D$  (it corresponds approximately to random packing), and the centers of large spherical particles are located at its vertices.

There is 0.5 large particle per cell. The shape of the hole between large particles can be approximated with duplex triangular pyramids having equal length of the base sides and height.

Small particles adjacent to large ones in the elementary cell usually have one contact each (similar to simple cubic packing), i.e., the density of part of the layer can be taken to be lower than the density of simple cubic packing (equal to 0.52) but higher than the threshold density of random packing (equal to 0.16). Assuming a density of the boundary-layer thickness of 0.34–0.40 to be the arithmetic mean of the cubic and threshold densities, we obtain  $V_b = 0.34$ .

The volume of the entire cell and the volume corresponding to the fine phase are  $0.433D^3$  and  $D^3/5.84$ . Using the

volume of the cell cavity, we determine the base side  $a$  of the pyramids approximating the cavity:

$$V_p = 2(0.433a^2h/3) = 0.2887a^3 = D^3/5.84.$$

Hence we obtain  $a = 0.84D$ .

The cell volume corresponding to the dense and loose portions of the fine-size phase is equal to  $0.288(a^3 - 1.5D_m\sqrt{3})^3$  and  $0.2887[a^3 - (a^3 - 1.5D_m\sqrt{3})^3]$ , respectively. The overall cell density (equal to the maximum relative density) is

$$V_{max} = V_{r,c} + (1 - V_{r,c})\{V_{r,m}(1 - 3D_m\sqrt{3}/a)^3 + 0.34[1 - (1 - 3D_m\sqrt{3}/a)^3]\},$$

or

$$V_{max} = V_{r,c} + (1 - V_{r,c})\{V_{r,m}(1 - 3D_m/D)^3 + 0.34[1 - (1 - 3D_m/D)^3]\}. \quad (1)$$

The optimum concentration of the large-size phase is

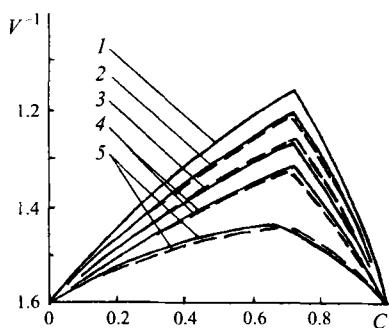
$$C_o = V_{r,c}/V_{max}. \quad (2)$$

The large particles comprising the dense part of the material correspond to  $(1 - C_o)/C_o$  small particles, i.e., the dense part of the packing corresponds to the share  $C/C_o$  of the material, where  $C$  is the volume concentration of the large particles. The dependence of the system density on the concentration of the large-size phase will take the form

$$V = V_{max} C/C_o + V_{r,m} (1 - C/C_o), \quad C < C_o. \quad (3)$$

In the case of an excess of the large particles (when the small particles can be freely located in the pores between them), the large particles form an additional volume lattice with a bulk density  $V_{r,c}$  (or a random-packing density equal to 0.63).

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**Fig. 1.** Dependence of the inverse relative density  $V$  of bidisperse powders of glass spheres on the concentration of the coarse phase  $C$  and the ratio of the particle sizes: 1) 0; 2) 1/16.5; 3) 1/6.5; 4) 1/4.8; 5) 1/3.4; 1, 2) computed from Eqs. (1) – (4); 4, 5) computed from Eqs. (3) – (6); solid curves) calculated data; dashed curves) experimental values [1].

The small particles contained in the dense part of the material correspond to  $C_o/(1 - C_o)$  small particles, i.e., the dense part of the packing corresponds to the share  $(1 - C)/(1 - C_o)$  of the material. Hence we obtain the following relationship between the system density and the concentration of the large-size phase:

$$V = V_{r,c}(C - C_o)/(1 - C_o) + V_{\max}(1 - C)/(1 - C_o), \quad C > C_o. \quad (4)$$

Let us determine the maximum theoretical density of freely poured bidisperse powders (with the ratio  $D_m/D \rightarrow 0$  and assuming that the small particles are distributed in the pores between the large particles with a completely random packing density, equal to 0.63) and the corresponding concentration of the large-size phase:

$$V_{\text{theor, max}} = 0.63 + 0.37 \times 0.63 = 0.863;$$

$$C_{o, D_m/D \rightarrow 0} = 0.63/0.863 = 0.73.$$

Hence the formulas for calculation of maximum theoretical density of freely poured bidisperse powders (with the ratio  $D_m/D \rightarrow 0$ ) will take the form

$$V_{o, D_m/D \rightarrow 0} = 1.183C + 0.63(1 - C/0.73), \quad 0 < C < 0.73;$$

$$V_{o, D_m/D \rightarrow 0} = 3.197(1 - C) + 0.63(C - 0.73)/0.27, \quad C > 0.73.$$

Calculations performed using Eqs. (1) – (4) indicated that they are applicable for the ratio  $D/D_m$  equal to no less

than 5. For lower ratios between the sizes of the large and small particles the following expressions should be used instead of Eqs. (1) and (2):

$$V_{\max} = V_{r,c} + (1 - V_{r,c}) \times \left\{ V_{r,m} \left( 1 - \frac{3D}{D_m} \right) + 0.34 \frac{D^2}{25D_m} \left[ 1 - \left( 1 - \frac{3D_m}{D} \right)^3 \right] \right\}; \quad (5)$$

$$C_o = 0.5 + \left( \frac{V_{r,c}}{V_{\max}} - 0.5 \right) \left( 1 - \frac{D_3}{125D_m^3} \right). \quad (6)$$

More complex expressions obtained in [3] can be used as well.

Figure 1 indicates dependences of the inverse relative density of bidisperse powders of glass spheres on the concentration of the large-size phase and the ratio of the particle sizes found from Eqs. (1) – (4) and (3) – (6), compared to the experimental data in [1]. It can be seen that the calculated data agree well with the experimental data.

The decrease in the packing density of particles observed as their average size decreases can be accounted for by the following approximate dependence:

$$\begin{aligned} V_r &= 1 - 0.1D^{-0.19} && \text{for } 10^{-3} > D > 6.5 \times 10^{-6} \text{ m;} \\ V_r &= 0.63 && \text{for } D > 10^{-3} \text{ m;} \\ V_r &= 0.97 && \text{for } D < 6.5 \times 10^{-3} \text{ m.} \end{aligned}$$

It should be noted that the shape and dimensions of the receiving container (mold) have an effect on the powder-batch density, and therefore, it is advisable to take these factors into account, as, for example, in [4].

Thus, rather simple expressions for determining the bulk density of bidisperse powder materials are derived. They are recommended for engineering calculations in the production of ceramic and composite materials.

## REFERENCES

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